τ-Information geometry and entropy

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τ-Information geometry is an information geometry based on dually coupled affine spaces: one is an affine space $\overset{\tau=s}{\mathcal{P}}$ constructed by finite positive measures which are mutually absolutely continuous and defined on some measurable space, and the other is an affine space $\overset{\tau=1-s}{\mathcal{P}}$ given by taking "Hölder conjugate" with respect to a parameter τ which controls an affine property of the space of finite positive measures $\mathcal{P}$. τ-Information geometry with $\tau = 1$ yields the same results from Amari-style information geometry, but it gives the different results for $\tau \neq 1$. The most prominent feature of τ-Information geometry is that the magnitude of a measure is changed depending on a translation in the affine spaces $\overset{\tau=s}{\mathcal{P}}$ and $\overset{\tau=1-s}{\mathcal{P}}$, and conversely a scale transformation induces a translation in the affine spaces.
Here we pay attention to an entropy. In the context of τ-Information geometry, the entropy is given in the form of a non-extensive entropy. However, in general, an entropy is affected by a normalization/scale, so a conformal entropy is naturally defined and used. Considering the change of a normalization caused by a translation as the change of a conformal entropy, we can restore the non-extensive conformal entropy to being an extensive conformal entropy. This is quite important because an entropy should be extensive in any physical contexts. The principle that an entropy should be extensive leads to the AdS/CFT correspondence or the holography principle, which makes complex and difficult situations occurring to calculate physical quantities simple and easy situations by introducing an extra codimension. Then, we introduce a coordinate for a scale transformation as an extra codimension, and consider the $(2r+1)$-dimensional space of natural coordinates $\left(\theta^1, \ldots, \theta^r\right)$ for characterizing a distribution, their dual coordinates $(\eta_1, \ldots, \eta_r)$, and the scale coordinate $\left(\theta^0\right)$. Now, if all of the coordinates are independent of each other, a contact structure is naturally arising. Furthermore, the corresponding Heisenberg group is also available so that the group structure of the Heisenberg group gives a Legendre transformation for potential functions.

[l] S. Goto, arXiv:1512.00950 (2016).
[2] M. Tanaka, J. of Phys.:Conf. S. **201**, 012007 (2010).
[3] S. Amari and A. Ohara, Entropy **13**, 1170 (2011).